

Pion correlations in Nuclear Matter

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Abstract

The saturation properties of the nuclear matter taking pion correlations into account is studied. We construct a Bogoliubov transformations for the pion pair operators and calculate the energy associated with the pion pairs. The pion dispersion relation is investigated. We next study the correlation energy due to one pion exchange in nuclear matter and neutron matter at random phase approximation using the generator coordinate method. The techniques of the charged pion correlations are discussed in the neutron matter calculations. We observe that there is no sign of the pion condensation in this model.

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I. INTRODUCTION

The understanding of the nuclear force at a microscopic level is an important problem since it shall be the basis for the nuclear matter and finite nuclei calculations. The interactions of nucleons which may arise as a residual interaction due to their substructure of quarks and gluons is technically not solvable. The alternative approach is to tackle the problem through meson interactions.

In recent years relativistic mean field theory (RMF)[1, 2] has been quite successful in describing nuclear matter and finite nuclei properties. The NN dynamics arises in this model from the exchange of a Lorentz scalar isoscalar meson, σ , which provides the mid range attraction and an isoscalar vector meson, ω , which provides the repulsion. Also, in this model the inclusion of the ρ - meson takes care of the neutron-proton asymmetry. With a small number of parameters, the RMF model reproduces the nuclear matter saturation and describes the bulk and the single particle properties for the finite nuclei reasonably well.

Despite the success of the RMF model, several open questions still remain unanswered. Firstly, the meson fields are classical and secondly the attractive part of the nuclear force is mediated through the hypothetical σ meson which could be an effect of multi-pion exchanges [3, 4, 5, 6, 7]. The σ can not be interpreted as the representation of a physical particle, since such a particle or resonant state remains still to be confirmed in the experiment. Thus on aesthetic as well as phenomenological ground, alternative approaches will add to our understandings.

The original Walecka model at the Hartree approximation does not contain a dynamical description of the pion fields. However the importance of the pions in NN dynamics can not be ignored. Realizing the essential role of pion in the description of the nuclear medium an alternative approach for the nuclear matter [4], deuteron [5] and for ^4He [6] has been developed. In this method, it has been studied the description of nuclear matter using pion pairs through a squeezed coherent state type of construction [4, 5, 6, 7]. This simulates the effects of the σ -meson and is a very natural quantum mechanical formalism for the classical fields.

The generator coordinate method (GCM) is a technique of great physical appeal which has been developed [8] to describe collective oscillations in nuclei. Besides being extensively used in nuclear structure physics, it often finds application in various other branches of physics

[9]. In this work we investigate the pion condensation problem in nuclear matter and neutron matter using the generator coordinate method. In a simplified model, the problem had been studied in Ref. [10] where a coherent state description for the pions was used. Using the gaussian overlap and harmonic approximation, the Hamiltonian may be diagonalized by a random phase approximation (RPA) like canonical transformation [11]. In this methodology, one can go beyond the coherent state description. Again the calculation can be carried out exactly, without further approximation. The present analysis is an extension of the mean field approach of Walecka where classical fields are replaced by quantum coherent states for the pion pairs. This report has also an advantage that the one pion exchange correlation contributions are considered at RPA level using similar Bogoliubov transformations. In the present model, we have observed no sign of the pion condensations in the RPA modes.

The outline of the paper follows. In section 2, we derive a pion nucleon Hamiltonian in a non-relativistic limit. We then construct a Bogoliubov transformations for the pion pair operators and calculate the energy associated with the pion pairs. In section 3, we calculate the correlation energies due to one pion exchange in nuclear matter and neutron matter at random phase approximation (RPA). Section 4 consists of the discussions of the saturation properties of nuclear matter, the pion dispersion relation in the medium and a concluding remarks.

II. FORMALISM

A. Non-relativistic Hamiltonian

The Lagrangian for the pion nucleon system is taken as

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - M + G\gamma_5\phi) \psi - \frac{1}{2} (\partial_\mu \varphi_i \partial^\mu \varphi_i - m^2 \varphi_i \varphi_i), \quad (1)$$

where $\psi = \begin{pmatrix} \psi_I \\ \psi_{II} \end{pmatrix}$ is the doublet of the nucleon field with mass M , φ_i 's are pion fields and $\phi = \tau_i \varphi_i$ represents the off-mass shell isospin triplet pion field with mass m . G is the pion-nucleon coupling constant. Repeated indices indicate summation.

The representations of γ matrices are

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}.$$

From the above Lagrangian, the equation of motions are

$$(E - M)\psi_I - (\vec{\sigma} \cdot \vec{p} + iG\phi)\psi_{II} = 0, \quad (2)$$

$$(E + M)\psi_{II} - (\vec{\sigma} \cdot \vec{p} - iG\phi)\psi_I = 0, \quad (3)$$

where, $E = i(\partial/\partial t)$ and $\vec{p} = i(\partial/\partial \vec{x})$. Eliminating the small component ψ_{II} from equation (2) and (3) we have

$$\left[(E^2 - M^2) - (E + M)(\vec{\sigma} \cdot \vec{p} + iG\phi)(E + M)^{-1}(\vec{\sigma} \cdot \vec{p} - iG\phi) \right] \psi_I = 0. \quad (4)$$

Equation (4) can be rewritten as

$$\left[E^2 - M^2 - p^2 + iG[(\vec{\sigma} \cdot \vec{p}), \phi] - G^2 \phi \cdot \phi \right] \psi_I = 0. \quad (5)$$

From equation (5), we can immediately identify the effective Hamiltonian for the nucleons as

$$\begin{aligned} \mathcal{H}_N &= \psi_I^\dagger(\vec{x}) \left[p^2 + M^2 - iG((\vec{\sigma} \cdot \vec{p})\phi) + G^2 \phi^2 \right]^{1/2} \psi_I(\vec{x}) \\ &\simeq \psi_I^\dagger(\vec{x}) \left[\epsilon_x - \frac{iG}{2\epsilon_x}((\vec{\sigma} \cdot \vec{p})\phi) + \frac{G^2}{2\epsilon_x} \phi^2 \right] \psi_I(\vec{x}) \\ &= \mathcal{H}_N^0(\mathbf{x}) + \mathcal{H}_{int}(\mathbf{x}), \end{aligned} \quad (6)$$

where the single particle nucleon energy operator ϵ_x is given by $\epsilon_x = (M^2 - \vec{\nabla}_x^2)^{1/2}$. In the non-relativistic assumption, we have to replace ϵ_x by M , when in a denominator and by $M + \frac{p^2}{2M}$ when not in a denominator. Now the effective Hamiltonian becomes

$$\mathcal{H}(\mathbf{x}) = \mathcal{H}_N^0(\mathbf{x}) + \mathcal{H}_{int}(\mathbf{x}) + \mathcal{H}_M(\mathbf{x}), \quad (7)$$

where the free nucleon part $\mathcal{H}_N^0(\mathbf{x})$ is given by

$$\mathcal{H}_N^0(\mathbf{x}) = \psi^\dagger(\mathbf{x}) \left(M + \frac{\nabla_x^2}{2M} \right) \psi(\mathbf{x}) , \quad (8)$$

the πN interaction Hamiltonian is provided by

$$\mathcal{H}_{int}(\mathbf{x}) = \psi^\dagger(\mathbf{x}) \left[-\frac{iG}{2M} ((\boldsymbol{\sigma} \cdot \mathbf{p}) \phi) + \frac{G^2}{2M} \phi^2 \right] \psi(\mathbf{x}) , \quad (9)$$

and the free meson part $\mathcal{H}_M(\mathbf{x})$ is defined as

$$\mathcal{H}_M(\mathbf{x}) = \frac{1}{2} \left[\dot{\varphi}_i^2 + (\nabla \varphi_i) \cdot (\nabla \varphi_i) + m^2 \varphi_i^2 \right] . \quad (10)$$

We expand the pion field operator $\varphi_i(\mathbf{x})$ in terms of the creation and annihilation operators of off-mass shell pions satisfying equal time algebra as

$$\varphi_i(\mathbf{x}) = \frac{1}{\sqrt{2\omega_x}} (a_i(\mathbf{x})^\dagger + a_i(\mathbf{x})), \quad \dot{\varphi}_i(\mathbf{x}) = i\sqrt{\frac{\omega_x}{2}} (a_i(\mathbf{x})^\dagger - a_i(\mathbf{x})) , \quad (11)$$

with energy $\omega_x = (m^2 - \nabla_x^2)^{1/2}$.

B. Correlation energy associated with two pions and Bogoliubov Transformation

The quadratic terms in the pion field in eq. (9) provide a isoscalar scalar interaction of nucleons and thus would simulate the effects of σ -mesons of the Walecka model.

A pion-pair creation operator given as

$$B^\dagger = \frac{1}{2} \sum_{\mathbf{k}} f_{\mathbf{k}} a_{\mathbf{k}i}^\dagger a_{-\mathbf{k}i}^\dagger , \quad (12)$$

is then constructed with the creation and annihilation operators in momentum space and the ansatz function $f(\mathbf{k})$. We then define the unitary transformation U as

$$U = e^{(B^\dagger - B)} \quad (13)$$

and note that U , operating on vacuum, creates an arbitrarily large number of scalar isospin

singlet pairs of pions corresponding to squeezed coherent states. We will show that this is the appropriate transformation to diagonalize the pion part of the Hamiltonian. The “pion dressing” of nuclear matter is then introduced through the state

$$|\Psi\rangle = U|0\rangle = e^{(B^\dagger - B)}|0\rangle. \quad (14)$$

We obtain

$$\tilde{a}_{\mathbf{k}i} = U^\dagger a_{\mathbf{k}i} U = (\cosh f_{\mathbf{k}}) a_{\mathbf{k}i} + (\sinh f_{\mathbf{k}}) a_{-\mathbf{k}i}^\dagger, \quad (15)$$

which is a Bogoliubov transformation. Here U is a unitary and hermitian operator. The psedo-pions $\tilde{a}_{\mathbf{k}i}$ are the results of the unitary transformation. It can also be easily checked that the operator $\tilde{a}_{\mathbf{k}i}$ satisfies the standard bosonic commutation relations:

$$[\tilde{a}_{\mathbf{k}i}, \tilde{a}_{\mathbf{k}'j}^\dagger] = \delta_{ij}\delta_{\mathbf{k},\mathbf{k}'}, \quad [\tilde{a}_{\mathbf{k}i}^\dagger, \tilde{a}_{\mathbf{k}'j}^\dagger] = [\tilde{a}_{\mathbf{k}i}, \tilde{a}_{\mathbf{k}'j}] = 0. \quad (16)$$

and also

$$\tilde{a}_{\mathbf{k}i}|\Psi\rangle = 0 \quad (17)$$

The reverse transformation:

$$a_{\mathbf{k}i} = (\cosh f_{\mathbf{k}}) \tilde{a}_{\mathbf{k}i} - (\sinh f_{\mathbf{k}}) \tilde{a}_{-\mathbf{k}i}^\dagger \equiv x_{\mathbf{k}} \tilde{a}_{\mathbf{k}i} - y_{\mathbf{k}} \tilde{a}_{-\mathbf{k}i}^\dagger. \quad (18)$$

In momentum space the effective Hamiltonian (7) may be re-written as

$$\begin{aligned} H \approx & \sum_{\mathbf{p}, \alpha \eta} \varepsilon_p c_{\mathbf{p}, \alpha \eta}^\dagger c_{\mathbf{p}, \alpha \eta} + \sum_{\mathbf{q}, j} \omega_{\mathbf{q}} a_{\mathbf{q}, j}^\dagger a_{\mathbf{q}, j} \\ & - \sum_{\mathbf{p}, j, \alpha \alpha' \eta \eta'} \frac{G}{2M\sqrt{\omega_{\mathbf{q}}V}} c_{\mathbf{p}+\mathbf{q}, \alpha \eta}^\dagger c_{\mathbf{p}, \alpha' \eta'} (i\sigma \cdot \mathbf{q})_{\alpha \alpha'} \tau_j (a_{\mathbf{q}, j} + a_{-\mathbf{q}, j}^\dagger) \\ & + \sum_{\mathbf{p}, j, \alpha \eta} \frac{G^2}{2M\omega_{\mathbf{q}}V} c_{\mathbf{p}, \alpha \eta}^\dagger c_{\mathbf{p}, \alpha \eta} (a_{\mathbf{q}, j}^\dagger a_{-\mathbf{q}, j}^\dagger + a_{\mathbf{q}, j} a_{-\mathbf{q}, j} + 2a_{\mathbf{q}, j}^\dagger a_{\mathbf{q}, j}). \end{aligned} \quad (19)$$

Here \mathbf{p} , α and η are respectively, the momentum, spin and iso-spin quantum numbers of the nucleon and \mathbf{q} , j are the momentum and isospin labels of the pion. $p = |\mathbf{p}|$, $q = |\mathbf{q}|$. $c_{\mathbf{p}, \alpha \eta}^\dagger$ is the creation operator for nucleon with momentum \mathbf{p} , spin α and isospin η . The contribution

of the quadratic term in the pion field coming from the above Hamiltonian as

$$\begin{aligned}
H_{2\pi} &= \sum_{\mathbf{q},j} \omega_{\mathbf{q}} a_{\mathbf{q},j}^{\dagger} a_{\mathbf{q},j} \\
&+ \sum_{\mathbf{p},\mathbf{q},j,\alpha\eta} \frac{G^2}{2M\omega_{\mathbf{q}}} c_{\mathbf{p},\alpha\eta}^{\dagger} c_{\mathbf{p},\alpha\eta} \left(a_{\mathbf{q},j}^{\dagger} a_{-\mathbf{q},j}^{\dagger} + a_{\mathbf{q},j} a_{-\mathbf{q},j} + 2a_{\mathbf{q},j}^{\dagger} a_{\mathbf{q},j} \right) \\
&= \sum_{\mathbf{q},j} \omega_{\mathbf{q}} a_{\mathbf{q},j}^{\dagger} a_{\mathbf{q},j} + \sum_{\mathbf{q},j} \frac{G^2 \rho}{2M\omega_{\mathbf{q}}} \left(a_{\mathbf{q},j}^{\dagger} a_{-\mathbf{q},j}^{\dagger} + a_{\mathbf{q},j} a_{-\mathbf{q},j} + 2a_{\mathbf{q},j}^{\dagger} a_{\mathbf{q},j} \right) \\
&= \sum_{\mathbf{q},j} \left(\omega_{\mathbf{q}} + \frac{G^2 \rho}{M\omega_{\mathbf{q}}} \right) a_{\mathbf{q},j}^{\dagger} a_{\mathbf{q},j} + \sum_{\mathbf{q},j} \frac{G^2 \rho}{2M\omega_{\mathbf{q}}} \left(a_{\mathbf{q},j}^{\dagger} a_{-\mathbf{q},j}^{\dagger} + a_{\mathbf{q},j} a_{-\mathbf{q},j} \right) \\
&= \sum_{\mathbf{q},j} \omega'_{\mathbf{q}} a_{\mathbf{q},j}^{\dagger} a_{\mathbf{q},j} + \sum_{\mathbf{q},j} \frac{g'}{2} \left(a_{\mathbf{q},j}^{\dagger} a_{-\mathbf{q},j}^{\dagger} + a_{\mathbf{q},j} a_{-\mathbf{q},j} \right)
\end{aligned} \tag{20}$$

where $\omega'_{\mathbf{q}} = \left(\omega_{\mathbf{q}} + \frac{G^2 \rho}{M\omega_{\mathbf{q}}} \right) = \omega_{\mathbf{q}} + g'$ with $g' = \frac{G^2 \rho}{M\omega_{\mathbf{q}}}$.

Now the equation of motion for the pions becomes

$$[H_{2\pi}, \tilde{a}_{\mathbf{q},j}^{\dagger}] = \tilde{\omega}_{\mathbf{q}} \tilde{a}_{\mathbf{q},j}^{\dagger}. \tag{21}$$

This gives

$$\omega'_{\mathbf{q}} x_{\mathbf{q}} a_{\mathbf{q},j}^{\dagger} + g' x_{\mathbf{q}} a_{\mathbf{q},j} - \omega'_{\mathbf{q}} y_{\mathbf{q}} a_{\mathbf{q},j} - g' y_{\mathbf{q}} a_{\mathbf{q},j}^{\dagger} = \tilde{\omega}_{\mathbf{q}} (x_{\mathbf{q}} a_{\mathbf{q},j} + y_{\mathbf{q}} a_{-\mathbf{q},j}^{\dagger}). \tag{22}$$

The characteristic equation is

$$\begin{vmatrix} (\omega'_{\mathbf{q}} - \tilde{\omega}_{\mathbf{q}}) & -g' \\ g' & -(\omega'_{\mathbf{q}} + \tilde{\omega}_{\mathbf{q}}) \end{vmatrix} = \tilde{\omega}_{\mathbf{q}}^2 - \omega'_{\mathbf{q}}{}^2 + g'^2 = 0, \tag{23}$$

which gives

$$\tilde{\omega}_{\mathbf{q}} = \sqrt{\omega'_{\mathbf{q}}{}^2 - g'^2} \quad \text{with} \quad x_{\mathbf{q}} = \sqrt{\frac{\omega'_{\mathbf{q}} + \tilde{\omega}_{\mathbf{q}}}{2\tilde{\omega}_{\mathbf{q}}}} \quad y_{\mathbf{q}} = \sqrt{\frac{\omega'_{\mathbf{q}} - \tilde{\omega}_{\mathbf{q}}}{2\tilde{\omega}_{\mathbf{q}}}}. \tag{24}$$

Now

$$H_{2\pi} = \sum_{\mathbf{q},j} \tilde{\omega}_{\mathbf{q}} \tilde{a}_{\mathbf{q},j}^{\dagger} \tilde{a}_{\mathbf{q},j} + \frac{3}{2} \sum_{\mathbf{q}} (\tilde{\omega}_{\mathbf{q}} - \omega'_{\mathbf{q}}). \tag{25}$$

We now have to include a term which corresponds to a phenomenological repulsion energy

between the pions of a “pair” in the above Hamiltonian $H_{2\pi}$ and is given by

$$H_m^R = A \sum_{\mathbf{q},j} e^{R_\pi^2 \mathbf{q}^2} a_{\mathbf{q},j}^\dagger a_{\mathbf{q},j} \quad (26)$$

where the two parameters A and R_π correspond to the strength and length scale, respectively, of the repulsion and will be determined phenomenologically. This term amounts to imposing a cut off on the momentum \mathbf{q} which accounts to the fact that momenta larger than k_f are not dynamically meaningful. With this repulsion term, we now have

$$\omega'_{\mathbf{q}} = \left(\omega_{\mathbf{q}} + A e^{R_\pi^2 \mathbf{q}^2} + \frac{G^2 \rho}{M \omega_{\mathbf{q}}} \right) = \omega_{\mathbf{q}} + A e^{R_\pi^2 \mathbf{q}^2} + g' \quad (27)$$

with

$$g' = \frac{G^2 \rho}{M \omega_{\mathbf{q}}}, \quad \omega_{\mathbf{q}} = \sqrt{\mathbf{q}^2 + m^2} \quad \text{and} \quad \tilde{\omega}_{\mathbf{q}} = \sqrt{\omega'^2_{\mathbf{q}} - g'^2}. \quad (28)$$

After transformation the Hamiltonian in equation (19) becomes

$$\begin{aligned} \tilde{H} \simeq & \sum_{\mathbf{p}, \alpha\eta} \varepsilon_{\mathbf{p}} c_{\mathbf{p}, \alpha\eta}^\dagger c_{\mathbf{p}, \alpha\eta} + \sum_{\mathbf{q}, j} \tilde{\omega}_{\mathbf{q}} \tilde{a}_{\mathbf{q}, j}^\dagger \tilde{a}_{\mathbf{q}, j} + \frac{3}{2} \sum_{\mathbf{q}} (\tilde{\omega}_{\mathbf{q}} - \omega'_{\mathbf{q}}) \\ & - \sum_{\mathbf{p}, \mathbf{q}, j, \alpha\alpha'\eta\eta'} \frac{g_{\mathbf{q}}}{\sqrt{\omega_{\mathbf{q}} V}} c_{\mathbf{p}+\mathbf{q}, \alpha\eta}^\dagger c_{\mathbf{p}, \alpha'\eta'} (i\boldsymbol{\sigma} \cdot \mathbf{q})_{\alpha\alpha'} \tau_j (\tilde{a}_{\mathbf{q}, j} + \tilde{a}_{-\mathbf{q}, j}^\dagger) \end{aligned} \quad (29)$$

where

$$g_{\mathbf{q}} = \frac{G (x_{\mathbf{q}} - y_{\mathbf{q}})}{2M}. \quad (30)$$

In the next section, we will consider RPA and calculate the correlation energy associated with one pion exchange for nuclear matter and neutron matter.

III. CORRELATION ENERGY ASSOCIATED WITH ONE PION EXCHANGE

A. nuclear matter

We consider a Slater determinant of plane waves

$$|\Phi\rangle = \prod_{\alpha\eta, |\mathbf{p}| \leq p_F} c_{\mathbf{p}, \alpha\eta}^\dagger |0\rangle, \quad (31)$$

with $|0\rangle$ is the absolute vacuum, p_F is the Fermi momentum and

$$c_{\mathbf{p},\alpha,\eta}|0\rangle = 0 . \quad (32)$$

Excitations with momentum transfer \mathbf{q} are coupled to excitations with momentum transfer $-\mathbf{q}$. Thus, the wave function $|\Psi\rangle$ which describes such excitations should read

$$|\Psi\rangle = \exp S|\Phi\rangle , \quad (33)$$

where

$$\begin{aligned} S_{\mathbf{q}j} &= U_{\mathbf{q}}\sqrt{\mathcal{N}_q} \sum_{\alpha,\alpha',\eta,\eta',\mathbf{p} \in \Omega_{\mathbf{q}}} \langle \alpha, \eta | (\sigma \cdot \mathbf{q}) \tau_j | \alpha', \eta' \rangle c_{\mathbf{p}+\mathbf{q},\alpha,\eta}^\dagger c_{\mathbf{p},\alpha',\eta'} \\ &+ U_{-\mathbf{q}}\sqrt{\mathcal{N}_q} \sum_{\alpha,\alpha',\eta,\eta',\mathbf{p} \in \Omega_{-\mathbf{q}}} \langle \alpha, \eta | -(\sigma \cdot \mathbf{q}) \tau_j | \alpha', \eta' \rangle c_{\mathbf{p}-\mathbf{q},\alpha,\eta}^\dagger c_{\mathbf{p},\alpha',\eta'} \\ &= U_{\mathbf{q}}B_{\mathbf{q},j}^\dagger + U_{-\mathbf{q}}B_{-\mathbf{q},j}^\dagger . \end{aligned} \quad (34)$$

In the above, \mathcal{N}_q is the normalization factor insuring

$$\mathcal{N}_q \sum_{\alpha\alpha'\eta\eta',\mathbf{p} \in \Omega_{\mathbf{q}}} |\langle \alpha, \eta | \vec{\sigma} \cdot \mathbf{q} \tau_j | \alpha', \eta' \rangle|^2 = 4\mathcal{N}_q \sum_{\mathbf{p} \in \Omega_{\mathbf{q}}} \mathbf{q}^2 = 1 \quad (35)$$

and the domain Ω_q is defined by $|\mathbf{p} + \mathbf{q}| > p_F$, $|\mathbf{p}| \leq p_F$. Only positive energy states are occupied. Here, $|\alpha, \eta\rangle$ denotes the spin iso-spin eigenstate, $\sigma_3|\alpha, \eta\rangle = \alpha|\alpha, \eta\rangle$ and $\tau_3|\alpha, \eta\rangle = \eta|\alpha, \eta\rangle$. The determination of the \mathcal{N}_q is now very simple. All we need is the volume of the intersection of 2 spheres of radius p_F , theirs centers being a distance q apart. With this assumptions, we have

$$\mathcal{N}_q^{-1} = \frac{V}{(2\pi)^3} 4\pi q^3 \left(p_F^2 - \frac{q^2}{12} \right) . \quad (36)$$

The transformed unperturbed Hamiltonian becomes

$$H_0 = \sum_{\mathbf{p},\alpha\eta} \varepsilon_p c_{\mathbf{p},\alpha\eta}^\dagger c_{\mathbf{p},\alpha\eta} + \sum_{\mathbf{q},j} \tilde{\omega}_{\mathbf{q}} \tilde{a}_{\mathbf{q},j}^\dagger \tilde{a}_{\mathbf{q},j} + \Delta , \quad \text{where} \quad \Delta = \frac{3}{2} \sum_{\mathbf{q}} (\tilde{\omega}_{\mathbf{q}} - \omega'_{\mathbf{q}}) . \quad (37)$$

The pion nucleon coupling reads,

$$H_{int} = - \sum_{\mathbf{p}\mathbf{q},j\alpha\alpha'\eta\eta'} \langle \alpha\eta | i(\vec{\sigma} \cdot \mathbf{q}) \tau_j | \alpha'\eta' \rangle \frac{g_{\mathbf{q}}}{\sqrt{\omega_{\mathbf{q}}V}} c_{\mathbf{p}+\mathbf{q},\alpha\eta}^\dagger c_{\mathbf{p},\alpha'\eta'} (\tilde{a}_{\mathbf{q},j} + \tilde{a}_{-\mathbf{q},j}^\dagger) . \quad (38)$$

In order to proceed, it is convenient to bosonize the Hamiltonian H , restricted to the subspace S . This is done by the replacement $S_{q,j} \rightarrow B_{q,j}$ satisfy boson commutation relations. The bosonized Hamiltonian reads as

$$H_B = E_0 + \sum_{\mathbf{q},j} \left(\varepsilon_q B_{\mathbf{q},j}^\dagger B_{\mathbf{q},j} - Q_q (B_{-\mathbf{q},j}^\dagger + B_{\mathbf{q},j}) (\tilde{a}_{\mathbf{q},j} + \tilde{a}_{-\mathbf{q},j}^\dagger) + \tilde{\omega}_q \tilde{a}_{\mathbf{q},j}^\dagger \tilde{a}_{\mathbf{q},j} \right) . \quad (39)$$

The parameters of H_B , namely ε_q and Q_q are fixed by expectation values such as

$$\langle \Phi | B_{\mathbf{q},j} H B_{\mathbf{q},j}^\dagger | \Phi \rangle = E_{HF} + \varepsilon_q, \quad \langle \Phi | \tilde{a}_{\mathbf{q},j} H B_{\mathbf{q},j}^\dagger | \Phi \rangle = Q_q,$$

and

$$\langle \Phi | \tilde{a}_{\mathbf{q},j} B_{-\mathbf{q},j} H | \Phi \rangle = Q_q. \quad (40)$$

The above Hamiltonian is diagonalized by a Bogoliubov transformation of the type

$$\Theta_{\mathbf{q},j}^{\dagger(n)} = x_1^{(n)} B_{\mathbf{q},j}^\dagger + x_2^{(n)} \tilde{a}_{\mathbf{q},j}^\dagger + y_1^{(n)} B_{-\mathbf{q},j} + y_2^{(n)} \tilde{a}_{-\mathbf{q},j}, \quad n = 1, 2 \quad (41)$$

which leads to excitation energies and the correlation energy.

In the above

$$\begin{aligned} \varepsilon_q &= 4 \mathcal{N}_q q^2 \sum_{\vec{p} \in \Omega_q} \frac{1}{2M} ((\vec{p} + \vec{q})^2 - p^2) = \frac{4\mathcal{N}_q q^2}{2M} \frac{V}{(2\pi)^3} \frac{4\pi q^2 p_F^3}{3} \\ &= \frac{1}{2M} \frac{4qp_F^3}{3(p_F^2 - q^2/12)} , \end{aligned} \quad (42)$$

and

$$Q_q = \sqrt{\frac{\mathcal{N}_q^{-1}}{2\omega_q V}} g_q . \quad (43)$$

The eigen frequencies are

$$\Omega_q^{(\pm)} = \frac{1}{\sqrt{2}} \sqrt{\varepsilon_q^2 + \tilde{\omega}_q^2 \pm \sqrt{(\varepsilon_q^2 - \tilde{\omega}_q^2)^2 + 16\varepsilon_q \tilde{\omega}_q Q_q^2}} . \quad (44)$$

The correlation energy becomes

$$E_{corr} = \frac{3}{2} \sum_{\vec{q}} \left(\Omega_q^{(+)} + \Omega_q^{(-)} - \varepsilon_q - \tilde{\omega}_q \right) . \quad (45)$$

B. neutron matter

The correlated Fermion wave function may be written, $\tau = -1$ for neutron and $\tau = 1$ for proton

$$|\Psi\rangle = \exp S |\Phi\rangle, \quad |\Phi\rangle = \prod_{\alpha, |\mathbf{p}| \leq p_F} c_{\mathbf{p}, \alpha, -1}^\dagger |0\rangle , \quad (46)$$

where the correlation operator reads

$$S = U_{\mathbf{q}} B_{\mathbf{q}, 0}^\dagger + U_{-\mathbf{q}} B_{-\mathbf{q}, 0}^\dagger + V_{\mathbf{q}} B_{\mathbf{q}, +}^\dagger + V_{-\mathbf{q}} B_{-\mathbf{q}, +}^\dagger , \quad (47)$$

with the quasi boson operators

$$B_{\mathbf{q}, 0}^\dagger = \sqrt{\mathcal{N}_q} \sum_{\alpha, \alpha', \mathbf{p} \in \Omega_{\mathbf{q}}} \langle \alpha, -1 | (\boldsymbol{\sigma} \cdot \mathbf{q}) \tau_0 | \alpha', -1 \rangle c_{\mathbf{p}+\mathbf{q}, \alpha, -1}^\dagger c_{\mathbf{p}, \alpha', -1} \quad (48)$$

$$B_{\mathbf{q}, +}^\dagger = \sqrt{\mathcal{N}'_q} \sum_{\alpha, \alpha', |\mathbf{p}| \leq p_F} \langle \alpha, 1 | (\boldsymbol{\sigma} \cdot \mathbf{q}) \tau_+ | \alpha', -1 \rangle c_{\mathbf{p}+\mathbf{q}, \alpha, 1}^\dagger c_{\mathbf{p}, \alpha', -1} , \quad (49)$$

where $\tau_0 = \tau_3$ and $\tau_+ = (\tau_1 + i\tau_2)/2$. Here, \mathcal{N}_q and \mathcal{N}'_q are normalization factors insuring

$$\mathcal{N}_q \sum_{\alpha, \alpha', \mathbf{p} \in \Omega_{-\mathbf{q}}} |\langle \alpha, -1 | \boldsymbol{\sigma} \cdot \mathbf{q} \tau_j | \alpha', -1 \rangle|^2 = 2\mathcal{N}_q \sum_{\mathbf{p} \in \Omega_{\mathbf{q}}} \mathbf{q}^2 = 1 , \quad (50)$$

$$\mathcal{N}'_q \sum_{\alpha, \alpha', |\mathbf{p}| \leq p_F} |\langle \alpha, 1 | \boldsymbol{\sigma} \cdot \mathbf{q} \tau_j | \alpha', -1 \rangle|^2 = 2\mathcal{N}'_q \sum_{\mathbf{p} \leq p_F} \mathbf{q}^2 = 1 . \quad (51)$$

As earlier the domain Ω_q is defined by $|\mathbf{p} + \mathbf{q}| > p_F$, $|\mathbf{p}| \leq p_F$. The determination of the normalization, \mathcal{N}_q and \mathcal{N}'_q , is now very simple. To compute \mathcal{N}_q , all we need is the volume of the intersection of two spheres of radius p_F , their centers being a distance q apart. We found

$$\mathcal{N}_q^{-1} = \frac{V}{(2\pi)^3} 2\pi q^3 \left(p_F^2 - \frac{q^2}{12} \right), \quad \mathcal{N}'_q^{-1} = \frac{V}{(2\pi)^3} \frac{8\pi}{3} q^2 p_F^3 . \quad (52)$$

The kinetic energy for the particle-hole pairs with momentum \mathbf{q} become

$$\begin{aligned}\varepsilon_q &= 2 \mathcal{N}_q q^2 \sum_{\vec{p} \in \Omega_q} \frac{1}{2M} ((\vec{p} + \vec{q})^2 - p^2) = \frac{2\mathcal{N}_q q^2}{2M} \frac{V}{(2\pi)^3} \frac{4\pi q^2 p_F^3}{3} \\ &= \frac{1}{2M} \frac{2qp_F^3}{3(p_F^2 - q^2/12)} ,\end{aligned}\tag{53}$$

$$\varepsilon'_q = 2 \mathcal{N}_q q^2 \sum_{\vec{p} \leq p_F} \frac{1}{2M} ((\vec{p} + \vec{q})^2 - p^2) = \frac{q^2}{2M} .\tag{54}$$

The pion nucleon interaction becomes

$$\begin{aligned}H_{int} &= - \sum_{\mathbf{p}\mathbf{q}, \alpha\alpha'\eta\eta'} \langle \alpha\eta | i(\vec{\sigma} \cdot \mathbf{q}) \tau_0 | \alpha'\eta' \rangle \frac{g_{\mathbf{q}}}{\sqrt{\omega_{\mathbf{q}}V}} c_{\mathbf{p}+\mathbf{q}, \alpha\eta}^\dagger c_{\mathbf{p}, \alpha'\eta'} (\tilde{a}_{\mathbf{q},0} + \tilde{a}_{-\mathbf{q},0}^\dagger) \\ &- \sum_{\mathbf{p}\mathbf{q}, \alpha\alpha'\eta\eta'} \langle \alpha\eta | i(\vec{\sigma} \cdot \mathbf{q}) \tau_+ | \alpha'\eta' \rangle \frac{g_{\mathbf{q}}}{\sqrt{\omega_{\mathbf{q}}V}} c_{\mathbf{p}+\mathbf{q}, \alpha\eta}^\dagger c_{\mathbf{p}, \alpha'\eta'} (\tilde{a}_{\mathbf{q},+} + \tilde{a}_{-\mathbf{q},-}^\dagger) \\ &- \sum_{\mathbf{p}\mathbf{q}, \alpha\alpha'\eta\eta'} \langle \alpha\eta | i(\vec{\sigma} \cdot \mathbf{q}) \tau_- | \alpha'\eta' \rangle \frac{g_{\mathbf{q}}}{\sqrt{\omega_{\mathbf{q}}V}} c_{\mathbf{p}+\mathbf{q}, \alpha\eta}^\dagger c_{\mathbf{p}, \alpha'\eta'} (\tilde{a}_{\mathbf{q},+} + \tilde{a}_{-\mathbf{q},-}^\dagger) .\end{aligned}\tag{55}$$

The effective bosonized Hamiltonian containing two pion exchange becomes

$$\begin{aligned}H_B &= E_0 + \sum_{\mathbf{q}} \left(\varepsilon_q B_{q,0}^\dagger B_{\mathbf{q},0} + \varepsilon'_q B_{q,+}^\dagger B_{\mathbf{q},+} + \tilde{\omega}_{\mathbf{q}} \sum_j \tilde{a}_{\mathbf{q},j}^\dagger \tilde{a}_{\mathbf{q},j} \right) \\ &- \sum_{\mathbf{q}} Q_q (B_{\mathbf{q},0}^\dagger + B_{\mathbf{q},0}) (\tilde{a}_{\mathbf{q},0} + \tilde{a}_{-\mathbf{q},0}^\dagger) \\ &- \sum_{\mathbf{q}} Q'_q (B_{\mathbf{q},+}^\dagger (\tilde{a}_{\mathbf{q},+} + \tilde{a}_{-\mathbf{q},-}^\dagger) + B_{\mathbf{q},+} (\tilde{a}_{-\mathbf{q},-} + \tilde{a}_{\mathbf{q},+}^\dagger)) ,\end{aligned}\tag{56}$$

where

$$Q_q = \sqrt{\frac{\mathcal{N}_q^{-1}}{2\omega_q V}} g_q, \quad Q'_q = \sqrt{\frac{\mathcal{N}'_q^{-1}}{2\omega_q V}} g_q .\tag{57}$$

The RPA equations are easily obtained. For the modes with charge,

$$\begin{aligned}&[H_B, (X_q B_{\mathbf{q},+}^\dagger + \zeta_q \tilde{a}_{\mathbf{q},+}^\dagger + \eta_q \tilde{a}_{-\mathbf{q},-})] \\ &= B_{\mathbf{q},+}^\dagger (X_q \varepsilon'_q + \zeta Q'_q - \eta_q Q'_q + \tilde{a}_{\mathbf{q},+}^\dagger (X_q Q'_q + \zeta_q \tilde{\omega}_q) + \tilde{a}_{-\mathbf{q},-} (X_q Q'_q - \eta_q \tilde{\omega}_q) \\ &= \Omega_q (X_q B_{\mathbf{q},+}^\dagger + x_q \tilde{a}_{\mathbf{q},+}^\dagger + y_q \tilde{a}_{-\mathbf{q},-}).\end{aligned}\tag{58}$$

The characteristic is a cubic and becomes

$$\begin{vmatrix} (\varepsilon'_q - \Omega_{\mathbf{q}}) & Q'_q & -Q'_q \\ Q'_q & \tilde{\omega}_{\mathbf{q}} - \Omega_q & 0 \\ Q'_q & 0 & -\tilde{\omega}_{\mathbf{q}} - \Omega_q \end{vmatrix} = -\Omega_q^3 + \varepsilon'_q \Omega_{\mathbf{q}}^2 + \tilde{\omega}_{\mathbf{q}}^2 \Omega_{\mathbf{q}} + 2Q_q'^2 \tilde{\omega}_{\mathbf{q}} - \varepsilon'_q \tilde{\omega}_{\mathbf{q}}^2 = 0 \quad (59)$$

where the solutions are

$$\begin{aligned} \Omega_q^{(1)} &= \frac{\varepsilon'}{3} + \frac{2^{1/3} \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)}{3 \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 + \sqrt{4 \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)^3 + \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 \right)^2} \right)^{1/3}} \\ &\quad - \frac{1}{3 \times 2^{1/3}} \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 + \sqrt{4 \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)^3 + \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 \right)^2} \right)^{1/3} \end{aligned} \quad (60)$$

$$\begin{aligned} \Omega_q^{(2)} &= \frac{\varepsilon'}{3} - \frac{(1 + i\sqrt{3}) \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)}{3 \times 2^{2/3} \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 + \sqrt{4 \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)^3 + \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 \right)^2} \right)^{1/3}} \\ &\quad + \frac{1}{6 \times 2^{1/3}} (1 - i\sqrt{3}) \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 + \sqrt{4 \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)^3 + \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 \right)^2} \right)^{1/3} \end{aligned} \quad (61)$$

$$\begin{aligned} \Omega_q^{(3)} &= \frac{\varepsilon'}{3} - \frac{(1 - i\sqrt{3}) \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)}{3 \times 2^{2/3} \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 + \sqrt{4 \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)^3 + \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 \right)^2} \right)^{1/3}} \\ &\quad + \frac{1}{6 \times 2^{1/3}} (1 + i\sqrt{3}) \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 + \sqrt{4 \left(-\varepsilon'^2 - 3\tilde{\omega}^2 \right)^3 + \left(-2\varepsilon'^3 - 54Q'^2 \tilde{\omega} + 18\varepsilon' \tilde{\omega}^2 \right)^2} \right)^{1/3} \end{aligned} \quad (62)$$

Similarly

$$[H_B, (Y_q B_{-\mathbf{q},+} + \tilde{\zeta}_q \tilde{a}_{-\mathbf{q},+}^\dagger + \tilde{\eta}_q \tilde{a}_{\mathbf{q},-})] = \Omega_q (Y_q B_{-\mathbf{q},+} + \tilde{\zeta}_q \tilde{a}_{-\mathbf{q},+}^\dagger + \tilde{\eta}_q \tilde{a}_{\mathbf{q},-}) . \quad (63)$$

This leads to a similar equation with Ω_q replaced by $-\Omega_q$, so that the eigenfrequencies occur in pairs, $\pm\Omega_q$. The correlation energy for the charged modes becomes

$$E'_{corr} = \frac{1}{2} \sum_q (|\Omega_q^{(1)}| + |\Omega_q^{(2)}| + |\Omega_q^{(3)}| - \varepsilon'_q - 2\tilde{\omega}_q) . \quad (64)$$

The eigen frequencies of uncharged modes are

$$\Omega_q^{(\pm)} = \frac{1}{\sqrt{2}} \sqrt{\varepsilon_q^2 + \tilde{\omega}_q^2 \pm \sqrt{(\varepsilon_q^2 - \tilde{\omega}_q^2)^2 + 16\varepsilon_q\tilde{\omega}_q Q_q^2}} . \quad (65)$$

The correlation energy becomes

$$E_{corr} = \frac{1}{2} \sum_{\vec{q}} \left(\Omega_q^{(+)} + \Omega_q^{(-)} - \varepsilon_q - \tilde{\omega}_q \right) . \quad (66)$$

which are given in earlier section.

IV. RESULTS AND DISCUSSION

We first proceed to describe the binding energy for nuclear matter. We obtain the free nucleon kinetic energy density

$$h_f = \langle \Phi | Tr[\hat{\rho}_N \mathcal{H}_N(\mathbf{x})] | \Phi \rangle = \frac{\gamma k_f^3}{6\pi^2} \left(M + \frac{3}{10} \frac{k_f^2}{M} \right) . \quad (67)$$

In the above equation, spin degeneracy factor $\gamma = 4$ (2) for nuclear matter (neutron matter) and , k_f represents the Fermi momenta of the nucleons. The Fermi momenta k_f and the nucleon densities are related by $k_f = (6\pi^2\rho/\gamma)^{\frac{1}{3}}$. It is well known that the short range

TABLE I: Parameters of the model obtained self consistently at saturation density.

a	R_π	λ_ω
(MeV)	(fm)	(fm ²)
14.58	1.45	3.07

interaction plays a crucial role in determining the saturation density which is mediated by the iso-scalar vector ω mesons. Here we introduce the energy of repulsion by the simple form [3, 4]

$$h_\omega = \lambda_\omega \rho^2, \quad (68)$$

where the parameter λ_ω is to be fixed using the saturation properties of nuclear matter as described in Ref.[7]. Thus we finally write down the binding energy per nucleon E_B of the symmetric nuclear matter (SNM):

$$E_B = \frac{E_0}{\rho} - M \quad (69)$$

where

$$E_0 = h_f + \frac{3}{2} \sum_q \tilde{\omega}_q + h_\omega . \quad (70)$$

In the above, E_0 is the energy density of nuclear matter or neutron matter without one pion correlation. The expression for E_0 contains the three model parameters a , R_π , and λ_ω as introduced in the earlier section. These parameters are determined self-consistently through the saturation properties of nuclear matter at saturation density $\rho_0 = 0.15 \text{ fm}^{-3}$ with and without the correlations. While pressure P vanishes at saturation density for symmetric nuclear matter, the values of binding energy per nucleon are chosen to be -16 MeV . In the numerical calculations, we have used the nucleon mass $M = 940 \text{ MeV}$, the pion masses $m = 140 \text{ MeV}$ and the omega meson mass, $m_\omega = 783 \text{ MeV}$, and the $\pi - N$ coupling constant $G^2/4\pi = 14.6$.

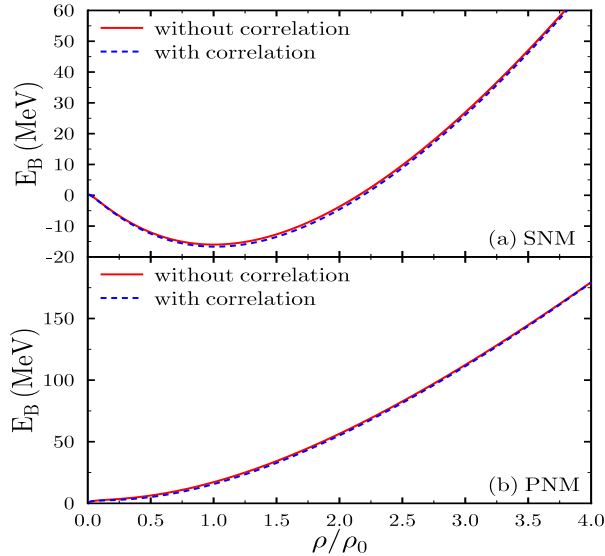


FIG. 1: Binding energy of symmetric nuclear matter (SNM) and pure neutron matter (PNM). The correlation is related to one-pion exchange

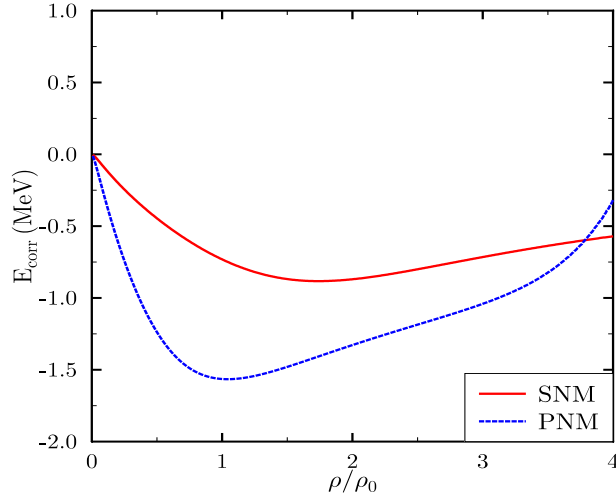


FIG. 2: The correlation energy from one-pion exchange in symmetric nuclear matter (SNM) and in pure neutron matter (PNM)

We now discuss the results obtained in our calculation. We first construct a Bogoliubov transformation for the pion pairs operators and calculate the energy associated with it. We next calculate the correlation energy due to the one pion exchange in nuclear matter and neutron matter at RPA using the generator coordinate method. The binding energy per nucleon E_B as a function of the density of the system is often referred as the nuclear equation of state (EOS). In figure 1, we present the EOS with and without correlation for the nuclear matter and for neutron matter. The correlation is related to one pion exchange. As expected, the binding energy for nuclear matter with and without correlation initially decreases with density and reaches a minimum at $\rho/\rho_0 = 1$ and then increases.

In figure 2, we show the variation of the correlation energy, E_{corr} as a function of density for symmetric nuclear matter (SNM) and for the pure neutron matter (PNM). The correlation energy initially decreases with density and then increases after the saturation density. The correlation energy for the neutron matter gives larger as compared to the nuclear matter at different densities.

Dispersion relation of modes with the quantum numbers of the pions in nuclear medium is an interesting aspect. In figure 3, we plot the dispersion relations arising from the two pion coherent states versus with the momentum at different densities. The increase of the pion dispersion relation for $k/k_f > 0.6$ is probably an artifact of the repulsion term of equation

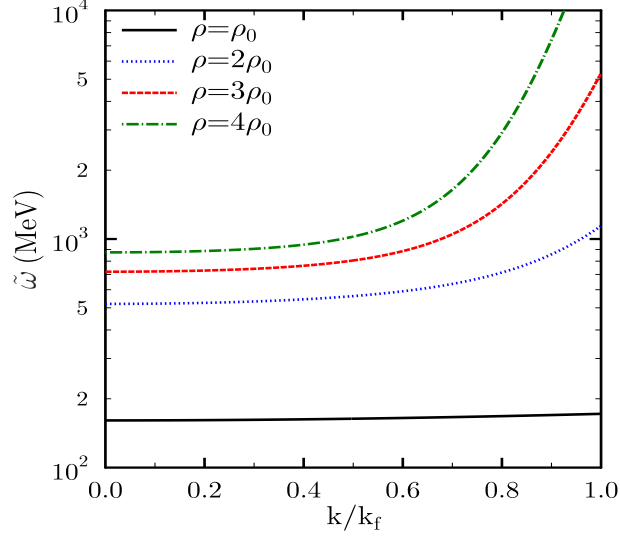


FIG. 3: The dispersion relation with the quantum number of the pions, $\tilde{\omega}$ for nuclear matter

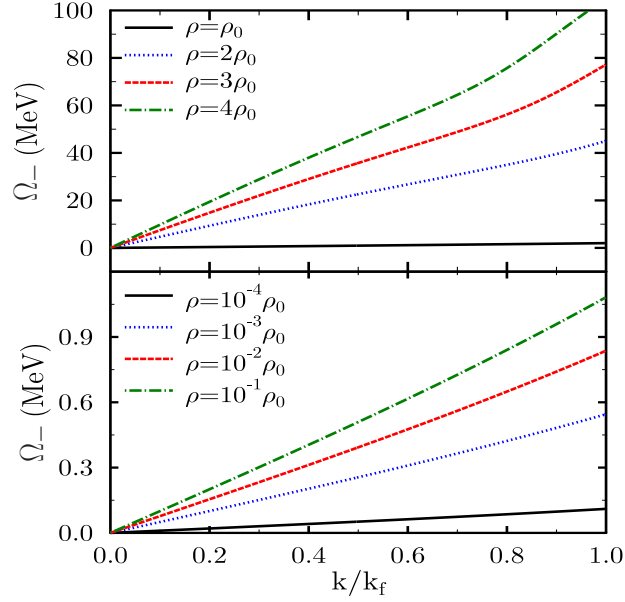


FIG. 4: The dispersion relation of the RPA mode with the quantum number of the pions, Ω_- , at high densities (upper panel) and for low densities (lower panel) for nuclear matter. This corresponds to zero sound modes.

(26).

For nuclear matter, we observed two RPA modes with quantum number of the pions, Ω_{\pm} . In figure 4, we have shown the dispersion relation for the RPA modes, Ω_- , versus momentum at different densities for nuclear matter. At $\rho = \rho_0$, Ω_- increases very slowly

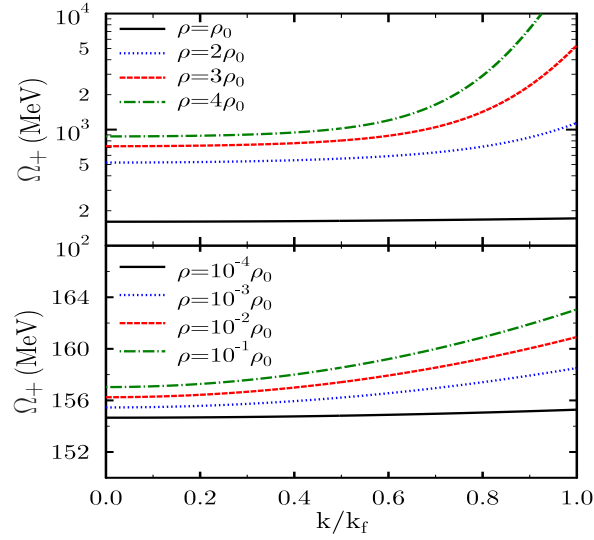


FIG. 5: The dispersion relation of the RPA mode with the quantum number of the pions, Ω_+ , at high densities (upper panel) and at low densities (lower panel) for nuclear matter. It is showing an increase with density of the effective mass of the pions.

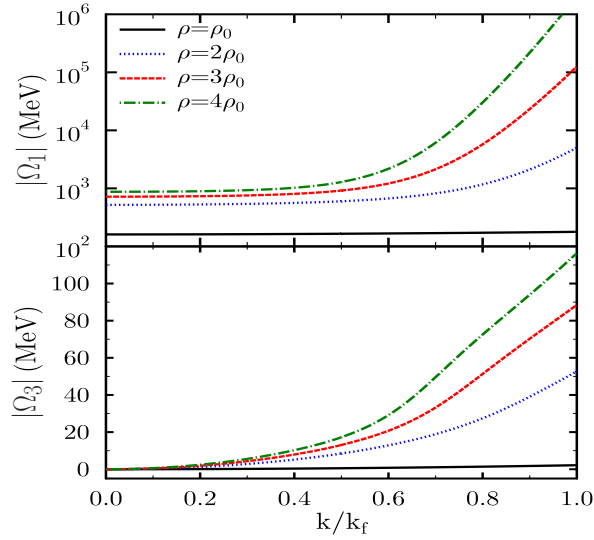


FIG. 6: The dispersion relation of the RPA mode with the quantum number of the pions, $|\Omega_1|$ and $|\Omega_3|$, for neutron matter

with momentum. However at $\rho = 4\rho_0$, it increases fast. In the lower panel of the figure 4, we have plotted Ω_- for smaller densities. At small densities, Ω_- increases monotonically with momentum k and corresponds to zero sound modes.

In figure 5, we have shown the dispersion relation for the RPA modes, Ω_+ , versus momentum at different densities for nuclear matter. It is found that the magnitude of the Ω_+ is

larger compared to Ω_- . In lower panel of the figure 5, we plotted Ω_+ for smaller densities, showing an increase with density of the effective mass of the pions.

We next study the RPA modes for the neutron matter. There are three modes for the charge pions. In figure 6, we plot the RPA frequencies versus momentum k for neutron matter. The $|\Omega_1|$ and $|\Omega_2|$ are equal. In the upper pannel, we show $|\Omega_1|$ versus momentum k . All the three RPA frequencies increase with density. In the lower panel, we plot $|\Omega_3|$ versus momentum k which corresponds to zero sound modes. It is found that there is no sign of pion condensation in higher densities with the RPA modes.

In conclusion, we have derived a pion nucleon Hamiltonian in a non-relativistic limit. We then have constructed a Bogoliubov transformations for the pion pair operators and calculate the energy associated with the pion pairs for the nuclear matter and neutron matter. This is an extension of the mean field approach of Walecka where the classical fields are replaced by the quantum coherent states for the pion pairs. We then calculated the correlation energies due to one pion exchange in nuclear matter and neutron matter at RPA using generator coordinate method. It is found that there is no sign of pion condensation in higher densities with the RPA modes.

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- [1] J.D. Walecka, Ann. Phys.(N.Y.) **83**, (1974) 491; B.D. Serot, J.D. Walecka, Int. J.Mod. Phys **E 6**, (1997) 515.
 - [2] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16**, (1986) 1.
 - [3] A. Mishra, H. Mishra and S.P. Misra, Int. J. Mod. Phys. **A 7**, (1990) 3391.
 - [4] H. Mishra, S.P. Misra, P.K. Panda and B. K. Parida, *Int. J. Mod. Phys.* **E 2**, (1992) 405.

- [5] P.K. Panda, S.P. Misra and R. Sahu, Phys. Rev. **C 45**, (1992) 2079.
- [6] P.K. Panda, S.K. Patra, S.P. Misra and R. Sahu, Int. J. Mod. Phys. **E 5**, (1996) 575.
- [7] S. Sarangi, P.K. Panda, S.K. Sahu and L. Maharana, Int. J. Mod. Phys. **B 22**, (2008) 4524,
S. Sarangi, P.K. Panda, S.K. Sahu and L. Maharana, to appear in Ind. J. Physics.
- [8] D.L. Hill and J.A. Wheeler, Phys. Rev. **89** (1953) 1102.
- [9] B. Johansson and J. da Provideência, Physica **B 94** (1978) 152.
- [10] J. da Provideência, Nucl. Phys. **A 290** (1977) 435.
- [11] P. Chattopadhyay and J. da Providência, Nucl. Phys. **A 370** (1981) 445.